

action of all the planets, except Jupiter, upon the Sun's surface. According to the values adopted in Norton's Astronomy, $(1 - \frac{2}{3}) \times (\sum md \div \sum m \delta, \text{♀}, \oplus, \text{♂}, \text{♂}_2, \text{♂}_3, \Psi) = 4.422$; $4.485 \div 4.422 = 1.015$.

The closeness of the harmony, pointed out by Kirkwood, between the Wolfian and Mercurial cycles, is very interesting. In some respects the position of Mercury is more nearly pivotal than that of either of the other planets. Its centre of explosive oscillation, relatively to the Sun, is near the limit of the Sun's possible atmosphere. $(\frac{4}{3} \times .3871)^{\frac{3}{2}} \times 365.2564 \div 25.187 = 1.036$. Its mean radius vector (.3871) is not much greater than the radius of the centre of oscillation (.3333) of the Earth's disturbing action upon the Sun. That centre of oscillation is, in its turn, near the centre of the counterpoising moments of inertia of the Sun and the planetary system. For $\sum m(d - .3333)^2 \div \sum m = 9.0776^2$; $759.46 \div 9.0776^2 = .3294^2$; $.3333 \div .3294 = 1.012$. It may be well to observe that this approximation is nearly identical with the one above noted (1.015) between the virtual centre of gravity of the Jupiter-disturbing planets, and the Sun-spot disturbance-perihelion.

In order to form an estimate of the extent, to which circumstances favoring the generation or disturbance of æthereal waves affect the amount of spotted surface, I would suggest a preliminary examination of observations with especial reference to the following planetary configurations:

1. When the Sun is near the linear centre of oscillation ($\frac{2}{3}$) of two planets in heliocentric opposition: *e. g.*, near the opposition of Mercury and Venus, especially if Mercury is about 60° from perihelion; of Venus and Mars, when Mars is about 40° from perihelion; of Jupiter and Saturn, when Saturn is near aphelion; of Saturn and Uranus.

2. When one of two planets in heliocentric conjunction is near a linear centre of oscillation of the longer radius vector: *e. g.*, near the conjunction of the Earth and Mars; of Mars and Mercury, if Mars is near perihelion and Mercury near aphelion; of Jupiter and Mars, if Mars is near aphelion; of Jupiter, Saturn and Uranus, (the centre of gravity of Jupiter and Saturn being at about $\frac{2}{3}$ of the distance from Uranus to the Sun); of Saturn and Neptune, when Saturn is near aphelion; of Uranus and Neptune.

3. When one of two planets in heliocentric conjunction is near a centre of explosive oscillation ($\frac{5}{8}$): *e. g.*, near the conjunction of Mercury and Venus, if Mercury is at somewhat more than its mean distance from the Sun; of Mercury and the Earth, if Mercury is near aphelion; of Mercury, Venus, and the Earth or Mars; of Jupiter and Saturn.

ÆTHEREAL OSCILLATION, THE PRIMORDIAL MATERIAL FORCE.

BY PLINY EARLE CHASE.

(Read before the American Philosophical Society, July 19, 1872.)

In any explosive or other analogous action along a given diameter, cardinal points occur at $\frac{1}{9}$ radius and $\frac{8}{9}$ radius (=the distance from

radial terminus to a centre of explosive oscillation), at $\frac{1}{3}$ and $\frac{2}{3}$ (=distance to a centre of linear oscillation), at $\frac{1}{2}$ (=distance to linear centre of gravity), and at $\frac{4}{9}$ and $\frac{5}{9}$ (=distance to a centre of reverse or direct explosive oscillation). The various possible combinations of these numbers favor phyllotactic and other systematic arrangements to such a degree, that the very wealth of planetary illustration becomes burdensome and perplexing, and the attempt to trace the development of our cosmical system to simple laws seems almost hopeless.

If the primitive disturbance is accompanied (as it must almost necessarily be), by rotation, the ratio 1 : 9 assumes special importance, inasmuch as it represents the proportionate distance of the centre of rotation from the rotating centre of explosive oscillation. Assuming r =Sun's radius, as our fundamental unit, we are at present unable to determine whether there is any permanent limit at $9r$, that point being within the sphere of solar atmospheric retardation; but I have shown in my communication of March 1, 1872, that there are important divisions,

Near 9^2r , the inner limit of the planetary system.....	α
Near 9^3r , the dividing mean between the intra- and the extra-asteroidal planets.....	β
Near 9^4r , the outer planetary limit.....	γ
The other ratios are exemplified by the positions of	
Saturn and Neptune, 1 : 3.....	δ
Earth and Mars, 2 : 3.....	ε
Saturn and Uranus, 1 : 2.....	ζ
Uranus and Neptune, from Saturn, 4 : 9.....	η
Uranus and Jupiter, from Neptune, 4 : 9.....	θ
Jupiter and Saturn, 5 : 9.....	ι

I do not find so much evidence of the marked cosmical influence of centres of linear oscillation, as I do of centres of direct and explosive oscillation. The positions $\frac{4r}{9}$ and $9 \times \frac{4r}{9}$, are both within the sphere of retardation; but at $9^2 \times \frac{4r}{9} = 36r$, we are near the centre of explosive oscillation between Mercury and the Sun, and also near the limit of the Sun's possible atmosphere,* which is probably not less than $35.59r$ (according to Spörer's estimate of the time of solar rotation), and not greater than $37.22r$ (according to Hornstein's estimate of the $26\frac{1}{3}$ day magnetic cycle). α

Near $9^3 \times \frac{4r}{9} = 324r$, we find the planet Mars..... λ

“ $9^4 \times \frac{4r}{9} = 2916r$, is a centre of explosive oscillation between Uranus and Saturn..... μ

* The limit of equal equatorial centrifugal and centripetal force.

The ratio $\frac{2}{9}$ is the unit factor for the explosive centres of oscillation of α , β , and γ .

The combined ratios of direct and reverse centres of oscillation, $\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$, determine the following interesting positions :

Near $9 \times \frac{20r}{81} = \frac{20r}{9}$, is the centre of gravity of the solar system, at heliocentric conjunction*..... ν

Near $9^2 \times \frac{20r}{81} = 20r$, is the centre of explosive oscillation, between the limit of solar retardation and the Sun's surface..... ξ

Near $9^3 \times \frac{20r}{81} = 180r$, is the centre of gravity of Mercury, Venus, and the Earth..... σ

Near $9^4 \times \frac{20r}{81} = 1620r$, is the centre of gravity of the planetary system. π

Near $9^5 \times \frac{20r}{81} = 14580r$, is a point which I will designate as the *reciprocal of gravity* for the solar system..... ρ

If the centre of explosive gyration from the Sun to Neptune be taken as a unit, and a point at a distance of 9 of those units $= 32805r$ be regarded as the seat of a new explosive action, the point ρ is near the centre of explosive gyration *towards the solar system*. If the centripetal is equal to the radial or centrifugal action, and if gravity is a resultant of æthereal oscillation, the square of the periodic time being 32 times as great as the square of the time of fall to an attracting centre, the modulus of light, relatively to the Sun, should be $32 \times \frac{4}{9} \times 32805r = 466560r$ ζ

The closeness of this estimate is shown by the following calculation :

log. v^2	10.527057†
“ g at Sun	1.219737
“ light mod.	11.307320
“ r	5.628452
“ mod. $\div r$	5.678868
∴ modulus of light $= 477384.6r$.	

The half-modulus is the height of virtual fall required, during orbital revolution at the Sun's surface, to acquire the velocity of light, if there were no æthereal resistance. Its relation to three cardinal positions in our system, is expressed in the following proportion :

$$\frac{1}{2} \text{ modulus} : \mathcal{U}'\text{'s rad. vec.} :: \oplus\text{'s rad. vec.} : \odot\text{'s rad.}$$

The approximate value of the Earth's radius vector, as thus determined, is $213.54r$ τ

* Whenever I have occasion, in the present paper, to speak of the centre of gravity of two or more planets, they will be considered as in heliocentric conjunction, and at their mean distances from the Sun.

† The values of the astronomical elements are taken from Norton's Astronomy.

The same modulus-ratio is also nearly equivalent to the ratio of the aggregate extra-asteroidal, to the aggregate intra-asteroidal planetary masses..... ν

If Mercury be regarded as having its mean distance at a direct explosive centre of oscillation, its perihelion distance is at the reverse explosive centre..... φ

The combination $\frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$, furnishes the following planetary positions.

Near $9^3 \times \frac{25r}{81} = 225r$, is the centre of gravity of the Earth and Mars. χ

Near $9^4 \times \frac{25r}{81} = 2025r$, is the orbit of Saturn..... ψ

The foregoing approximations are embodied in the following synopsis :

TABLE I.

Centrifugal Evidences of Planetary Explosive Oscillation.

	(A) Theoretical.	(B) Observed.	(B-A) ÷ A.	Basis.
α	81.	83.17	.027	9^2
β	729.	734.93	.008	9^3
γ	6561.	6453.73	-.016	9^4
δ	.3333	.3149	-.055	$\frac{1}{3}$
ε	.6667	.6563	-.016	$\frac{2}{3}$
ζ	.5	.4973	-.005	$\frac{1}{2}$
η	.4444	.4705	.059	$\frac{4}{9}$
θ	.4444	.4371	-.016	$\frac{4}{9}$
ι	.5556	.5454	-.018	$\frac{5}{9}$
κ	36.	36.96	.027	$\frac{4}{9} \times 9^2$
λ	324.	326.59	.008	$\frac{4}{9} \times 9^3$
μ	2916.	2970.43	.019	$\frac{4}{9} \times 9^4$
ν	2.2222	2.17	-.025	$\frac{4}{9} \times \frac{5}{9} \times 9$
ξ	20.	20.43	.022	$\frac{4}{8} \times \frac{5}{8} \times 9^2$
$\omicron$	180.	182.83	.016	$\frac{4}{9} \times \frac{5}{9} \times 9^3$
π	1620.	1625.69	.004	$\frac{4}{9} \times \frac{5}{9} \times 9^4$
ρ	14580.	14341.64	-.016	$\frac{4}{9} \times \frac{5}{9} \times 9^5$
ς	466560.	477384.6	.023	$32 \times \rho$
τ	213.54	214.86	.006	ς
υ	213.54	206.29	-.034	ς
φ	.8	.7945	-.007	$\frac{4}{9} \div \frac{5}{9}$
χ	225.	225.46	.002	$\frac{5}{9} \times \frac{5}{9} \times 9^3$
ψ	2025.	2049.51	.01	$\frac{5}{9} \times \frac{5}{9} \times 9^4$

The importance of combining the centres of direct and reverse explosive gyration, in forming the units of the series which is to contain the modulus of equal universal action and reaction, is obvious.

Reversing our procedure, and commencing at the centre of oscillation of the reciprocal of gravity (α'), we find the following relationships :

$\frac{2}{3} \alpha'$	is near the mean position of Neptune.....	β'
$\frac{5}{9} \alpha'$	" the c. grav. of Neptune and Uranus.....	γ'
$\frac{5}{9} \gamma'$	" " of Neptune, Uranus and Saturn.....	δ'
$\frac{5}{9} \delta'$	" " of the four outer planets.....	ε'
$\frac{2}{3} \varepsilon'$	" " mean position of Jupiter.....	ζ'

The centre of linear oscillation, therefore, appears to have had an important influence in determining the positions of Neptune and Jupiter, and the centre of explosive oscillation in fixing the places of the intermediate planets. Neptune's position is also near the reverse centre of explosive oscillation of the reciprocal of gravity, $\frac{2}{3}$ of $\frac{2}{3}$ being equivalent to $\frac{4}{9}$.

From the excess of the observed over the theoretical value of the modulus of light (ρ), in the foregoing table, as well as from the fact that the theoretical modulus best satisfies the hypothesis of explosive positions in the outer planetary system, I infer that the mean velocity of light between Neptune and the Sun, is retarded a little more than one per cent. by the condensation of æther towards the Sun.

$\frac{5}{9} \zeta'$	is near the centre of gravity of the principal asteroids, and also near the centre of explosive oscillation from Cybele to Flora.....	η'
$\frac{5}{9} \eta'$	is near the mean position of Mars.....	θ'
$\frac{5}{9} \theta'$	is near the centre of gravity of the three interior planets.....	i'
$\frac{5}{9} i'$	is near the linear centre of oscillation of Venus.....	z'
$\frac{5}{9} z'$	is near Mercury's perihelion.....	λ'
$\frac{5}{9} \lambda'$	is near the explosive centre of gyration, which limits solar atmospheric retardation.....	μ'

This second series of coincidences is grouped in the following table :

TABLE II.
Centripetal Evidences of Explosive Oscillation.

	(A) Theoretical.	(B) Observed.	(A-B) ÷ A	Basis.
α'	9720			$\frac{1}{43}$ modulus.
β'	6480	6453.72	.004	$\frac{2}{3} \alpha'$
γ'	5400	5429.62	-.005	$\frac{5}{9} \alpha'$
δ'	3000	2894.57	.035	$\frac{5}{9} \gamma'$
ε'	1633.33	1632.69	.000	$\frac{5}{9} \delta'$
ζ'	1088.89	1117.87	-.027	$\frac{2}{3} \varepsilon'$
η'	604.94	588.07*	.028	$\frac{5}{9} \zeta'$
	"	589.41	.026	"
θ'	336.08†	326.59	.029	$\frac{5}{9}$
i'	186.71	182.83	.021	$\frac{5}{9} \theta'$
z'	103.73	103.61	.001	$\frac{5}{9} i'$
λ'	57.63	56.07	.027	$\frac{5}{9} z'$
μ'	32.02	32.36†	-.011	$\frac{5}{9} \lambda'$

* Mean radius vector of Juno, Ceres, and Pallas.

† $\frac{8}{9}$ of 36.4 ; 36.4 being the mean of the two extreme values, derived from Sporer's and Hornstein's estimates.

$$\rho' = \frac{5}{8} \times \left(\frac{3}{2}\right)^9 \times r$$

$$\rho' \div r = \text{modulus of light} \div \rho = T^2 \div t^2$$

There are, therefore, NINE principal planetary orbital divisions, determined mainly by centres of explosive gyration; NINE successive centres of linear oscillation between the modulus of light and the reciprocal of gravity; NINE similar centres between the limit of the Sun's possible atmosphere and the Sun's surface; and finally, NINE successive powers of NINE, that exert important centrifugal determinations. For we find:

At $9^{-3} r$ the fulcrum of the solar system, if the planetary masses were concentrated in a point at r .

At $9^{-2} r$ is the seat of an explosive oscillation, which would tend to detach a nebulous ring, with a mass $9^{-3} m$, provided the centre of oscillation of the ring were at the present geometrical planetary mean, $9^{-3} r$. For, in order that the separation should begin, the velocity of nebular rotation at the surface of the central body should bear such proportion to the mean velocity of revolution in the ring as the radius of the central mass to the radius of the ring's centre of oscillation, and as the velocity of virtual revolution at the central surface, to the like velocity at the seat of explosion, $(1 \div 81^{\frac{3}{2}})$

At $9^{-1} r$, a planetary mass which was in nebulous condition in the above case, having acquired sufficient central condensation for the detachment of an interior planet, would have its fulcrum at the Sun's surface, if its centre of gravity were at the distance of Mercury. The combined effect of these two primitive hypothetical breaks is perhaps recorded in the solar rotation, which occupies nearly, if not *precisely*, $\frac{8}{9} \times \frac{1}{3} \times 729$, $\times$ the planetary orbital time at the Sun's surface.

At $9^0 r$ is the Sun's surface, the probable seat of many of the explosions which have lately attracted so much attention.

At NINE r , is the first theoretical exterior limit of explosion, in which the Sun is central and its opposite surfaces are centres of alternate direct and reverse explosive oscillation. Here is, perhaps, the limit of an important solar envelope.

Near NINE times the first exterior limit, or $9^2 r$, is the orbit of Mercury.

Near NINE times the second exterior limit, or $9^3 r$, is the asteroidal limit.

Near NINE times the third exterior limit, or $9^4 r$, is the orbit of Neptune.

Near NINE times the fourth exterior limit, or $9^5 r$, is the centre of Sunward explosive oscillation which determines the reciprocal of gravity.

Such considerations, conjoined with the observed increase of spotted surface on the side of the Sun next the Earth and most remote from other disturbing planets,* seem to furnish a clue to the solution of the Sun-spot mystery. Even if all the planets were so arranged as to be either in mutual heliocentric conjunction or opposition, their combined

* De La Rue, Stewart and Loewy. *Proc. Roy. Soc.*, xx., 210, *sqq.*

influence would not suffice to raise a tide of a quarter-inch at the Sun's surface. But variations of inertia ($m d^2$) and of power to produce local oscillations ($\frac{m}{d^2}$) may be much more important, in their influence on

fluid bodies, than differential tidal variations ($\frac{m}{d^3}$). At the very outset of my cosmical investigations (*ante*, ix., 287), I gave a method for estimating the Sun's distance by comparing the atmospheric inertia relatively to the Earth, with the same inertia relatively to the Sun. The remarkable coincidence of the disturbing focus of the Wolfian cycle with the centre of linear oscillation in the planetary system, seems to render it probable that solar explosions and spots are also determined by laws of inertia and local oscillation.

ON BATHMODON, AN EXTINCT GENUS OF UNGULATES.

BY EDWARD D. COPE.

(*Read before the American Philosophical Society, Feb. 16, 1872.*)

The present form embraces some of the largest Perissodactyles, or odd-toed Ungulata, of our Tertiary Strata. It is represented by remains of two species, which include portions of the cranium, with teeth and fragments of jaws; vertebrae, fragments of scapular and pelvic arches and bones of the limbs. The distal end of the tibia is wanting, but that of the fibula indicates an odd-toed animal, and the third trochanter on the exterior ridge of the femur confirms the reference.

There are probably four superior molars, though three only are preserved. Two premolars only remain of the superior species, but the fragment of ramus mandibuli, referred to the same species, exhibits four premolars; from a consideration of the sizes of the superior premolars it is probable that there were four of these also. There are three strong incisors in each premaxillary. No canine tooth is preserved, but the posterior suture of the premaxillary bone is so wide as to point to an equally stout anterior part of the maxillary, fitted to support such a tooth. The dental series increases regularly in size, from before backwards, the last being a little larger than the penultimate. The crowns of the molars exhibit on the outer margin a single, acutely-angled crescent, directed inwards, with a conic lobe alongside of, and anterior to its base, representing a second external crescent. The crescent lobe proper is large and very obliquely directed, so that its external face is almost horizontal. The apex of its companion cone is continuous with its posterior margin, so as to be undistinguishable from it in some cases. The inner crescents are represented by a wide angular ridge, which is at a lower level than the exterior, and is little or not developed on the posterior side of the crown.